2nd Congress of Greek Mathematicians

Special Session: *Differential Equations*

Organizers*: Drosos Gintides, Georgia Karali, Vassilios Papageorgiou*

Tuesday 5/7/2022

16:30 – 17:10	N. Alikakos (University of Athens) *Analogies between minimal partitions and entire solutions of the vector Allen-Cahn equation*
17:15 – 17:45	N. Roidos (University of Patras) *The fractional porous medium equation on manifolds with conical singularities*
17:45 – 18:00	Break
18:00 – 18:30	D. Antonopoulou (Chester University, UK) *The monotonicity formula of the stochastic mean curvature flow of graphs*
18:30- 19:00	K. Dareiotis (University of Leeds, UK) *Regularisation of differential equations by noise*
19:00 – 19:30	A. Logioti (University of Bonn, Germany) *A non-local free boundary problem arising in a model of cell polarization*
19:30 – 20:00	K. Tzirakis (IACM/FORTH & University of Crete) *Layer dynamics for the one-dimensional ε-dependent Cahn-Hilliard/Allen-Cahn equation*

Wednesday 6/7/2022

16:30 – 17:10	A. Tzavaras (KAUST, Applied Mathematics and Computational Science) *Existence theory and propagation of oscillations for the system of viscoelasticity of strain-rate type*
17:15 – 17:45	N. Karachalios (University of Thessaly) *The Closeness of the Ablowitz-Ladik Lattice to non-integrable DNLS Lattices*
17:45 – 18:15	Break
18:15 – 18:45	A. Charalambopoulos (NTUA) *The inverse acoustic scattering problem via stochastic implementation*

18:45- 19:15	G. Sakellaris (Aristotle University of Thessaloniki) *Scale invariant regularity estimates for second order elliptic equations with lower order coefficients in optimal spaces*
19:15 – 19:45	L. Mindrinos (NTUA) *A numerical solution for various lateral Cauchy problems*

Thursday 7/7/2022

16:30 – 17:10	G. Dassios (University of Patras) *Simple solutions in anisotropic elasticity*
17:15 – 17:45	G. Moschidis (Princeton University, USA) *The emergence of weak turbulence in general relativity*
17:45 – 18:15	Break
18:15 – 18:45	I. Karafyllis (NTUA) *Feedback control of the 1d viscous Saint-Venant model*
18:45- 19:15	V. Bitsouni (University of Athens) *An inequality regarding completely monotone functions*
19:15 – 19:45	K. Gkikas (University of Athens) *Quasilinear elliptic equations involving absorption terms and measure data*

Titles and Abstracts

40 minute talks

Nicholas Alikakos (University of Athens)

Analogies between minimal partitions and entire solutions of the vector Allen-Cahn equation

There is a well-established analogy between minimal surfaces and the scalar two-phase Allen-Cahn equation, discovered by De Giorgi in the 70s, which has been thoroughly worked out in the last few years. The basic object, in this case, is a plane modeled by the solution of a simple ODE. On the other hand in the vector case, which is relevant for three or more coexisting phases, the basic objects are PDE solutions and are analogous to minimal partitions. These are fully understood in the presence of symmetry (general reflection point group), but very little in the absence of symmetry. In the lecture, I will explain the case of symmetry and describe some recent work relevant to the general case.

George Dassios (University of Patras)

Simple solutions in anisotropic elasticity

The classical homogeneous and isotropic elastostatics is governed by a linear vector differential equation of the second order where the constant coefficients are expressed in terms of 2 independent constants, known as the Lame parameters. On the other hand, when the space is anisotropic, this equation involves coefficients depending on 21 independent constants, known as the elastic constants. As a consequence, the corresponding equation, which has 4 vector terms in the isotropic case, in the presence of anisotropy, the vector terms of the equation become 27. This makes any attempt to generate analytic solutions extremely hard if not impossible. However, it seems that it is possible to generate interesting closed form solutions first in Cartesian form, and then in any other system, by applying simple algebraic manipulations. Low degree solutions in special anisotropies can be easily obtained, but for higher degree solutions and more general anisotropies the use of a computational machine is very helpful. An interesting aspect of this program is the particular pattern in which the algebraic coefficients of a polynomial solution interweave with the elasticity constants that specify the anisotropic structure of the space. As an example of this technique the special case of a material with cubic anisotropy will be worked out.

Athanasios Tzavaras (KAUST, Applied Mathematics and Computational Science)

Existence theory and propagation of oscillations for the system of viscoelasticity of strain-rate type

I will review the existence and uniqueness theory for viscoelasticity of Kelvin-Voigt type with non-convex stored energies. The analysis is based on propagation of H^1-regularity for the deformation gradient of weak solutions in two and three dimensions assuming that the stored energy satisfies the Andrews-Ball condition, in particular allowing for non-monotone stresses. it turns out that weak solutions with deformation gradient in H^1 are in fact unique, providing a striking analogy to corresponding results in the theory of 2D Euler equations with bounded vorticity. On the opposite direction, while there is still existence of weak solution for initial data in L^2, there can be propagation of oscillations of the deformation gradient. A counterexample indicates that for non-monotone stress-strain relations in 1-d initial oscilla- tions of the strain lead to solutions with sustained oscillations. Similar phenomena appear in several space dimensions associated with lack of rank-one convexity of the stored energy. (Joint work with K. Koumatos (U. of Sussex), C. Lattanzio and S. Spirito (U. of L' Aquila)).

25 minute talks

Dimitra Antonopoulou (Chester University, UK)

The monotonicity formula of the stochastic mean curvature flow of graphs

We consider the motion of normal graphs driven by mean curvature flow with a uniform in space white noise of Stratonovich type, and prove that it is the limit problem of a mildly stochastically perturbed flow along the normal vector. We derive in a geometric manner the elegant Stochastic Monotonicity Formula and its weak formulation on the Stratonovich limit which acts as the maximum principle over geometric equations. Then, by use of the weak formulation, under the assumption that the locally existing stochastic solutions are global for any finite time, we estimate the second fundamental form, and prove a.s. decay for its higher order derivatives at infinite times. (Joint with: G. Karali, A.N.K. Yip).

Vasiliki Bitsouni (University of Athens)

An inequality regarding completely monotone functions

We present an inequality which combines the concept of completely monotone functions with the theory of divided differences. It emerges from the study of the blow up time of certain solutions of the Cauchy problem for the generalised one-dimensional model of Population Ecology. A particular case of this inequality is a multidimensional analogue of the basic inequality $1 + x \leq e^x$, for an abstract non negative real number x. Joint work with Nikolaos Gialelis (National and Kapodistrian University of Athens) and Dan-Stefan Marinescu (National College “Iancu de Hunedoara”).

Antonios Charalambopoulos (NTUA)

The inverse acoustic scattering problem via stochastic implementation

A novel stochastic method has been recently developed and investigated in order to face the time-reduced inverse scattering problem, governed by Helmholtz equation, outside connected or disconnected obstacles supporting boundary conditions of Dirichlet type. On the basis of the stochastic analysis, a series of efficient and alternative stochastic representations of the scattering field have been formulated. These novel representations constitute conceptually the probabilistic analogue of the well-known deterministic integral representations involving the Green’s functions, and so merit special importance. Their advantage lies on their intrinsic probabilistic nature, allowing to solve the direct and inverse scattering problem in the realm of local methods, which are strongly preferable in comparison with the traditional global ones. The aforementioned locality reflects the ability to handle the scattering field only in small bounded portions of the scattering medium, by monitoring suitable stochastic processes, confined in narrow sub-regions where data are available. Especially in the realm of the inverse scattering problem, two different schemes are proposed facing reconstruction from the far field and near field data, respectively. The crucial characteristic of the inversion is that the reconstruction is fulfilled through stochastic experiments, taking place in the interior of conical regions whose base belong to the data region, while their vertices detect appropriately the supporting surfaces of the sought scatterers.

Konstantinos Dareiotis (University of Leeds, UK)

Regularisation of differential equations by noise

In this talk we will discuss about stochastic differential equations whose deterministic counterparts are not even well posed. We will see how the noise helps both towards their well-posedness and their numerical approximation. We will see equations regularised by Markovian and non-Markovian noises. If time permits, we will also discuss about regularisation of PDEs by noise. The main tool for our analysis is the recently obtained “Stochastic Sewing Lemma”. The talk is based on joint works with O. Butkovsky, M. Gerencsér, and K. Lê.

Konstantinos Gkikas (University of Athens)

Quasilinear elliptic equations involving absorption terms and measure data

Let $1 < p < N$ and $\Omega \subset R^N$ be an open bounded domain. We study the existence of solutions to equation (E_+) $-\Delta_p u + g(u)\sigma = \mu$ in Ω, where $g \in C(R)$ is a nondecreasing function, μ is a bounded Radon measure on Ω and σ is a nonnegative Radon measure on R^N. We show that if σ belongs to some Morrey space of signed measures, then we may investigate the existence of solutions to equation (E_+) in the framework of renormalized solutions. Furthermore, imposing a subcritical integral condition on g, we prove that equation (E_+) admits a renormalized solution for any bounded Radon measure μ. When $g(t) = |t|^{q-1}t$ with $q > p - 1$, we give various sufficient conditions for the existence of renormalized solutions to (E_+). These sufficient conditions are expressed in terms of Bessel capacities.

The research project was supported by the Hellenic Foundation for Research and Innovation (H.F.R.I.) under the "2nd Call for H.F.R.I. Research Projects to support Post-Doctoral Re- searchers" (Project Number: 59).

Iasson Karafyllis (NTUA)

Feedback control of the 1d viscous Saint-Venant model

This work studies the feedback stabilization problem of the motion of a tank that contains an incompressible, Newtonian, viscous liquid. The control input is the force applied on the tank and the overall system consists of two nonlinear Partial Differential Equations (PDEs) and two Ordinary Differential Equations. Moreover, a spill-free condition is required to hold. By applying the Control Lyapunov Functional methodology, we achieve semi-global stabilization of the liquid and the tank by means of a simple state feedback law. The closed-loop system exhibits an exponential convergence rate to the desired equilibrium point. The proposed stabilizing state feedback law does not require measurement of the liquid level and velocity profiles inside the tank and simply requires measurements of: (i) the tank position error and tank velocity, (ii) the total momentum of the liquid, and (iii) the liquid levels at the tank walls. Based on the constructed state feedback laws, we also solve the output-feedback stabilization problem for a tank with a liquid modeled by the viscous Saint-Venant PDE system. In this case the measurements are the tank position and the liquid level at the tank walls. The control scheme is a combination of a state feedback law with functional observers for the tank velocity and the liquid momentum. Again, exponential convergence of the closed-loop system to the desired equilibrium point is achieved. An algorithm is provided that guarantees that a robotic arm can move a glass of water to a pre-specified position no matter how full the glass is, without spilling water out of the glass, without residual end point sloshing and without measuring the water momentum and the glass velocity. Finally, the efficiency of the proposed feedback laws is validated by numerical examples, obtained by using a novel finite-difference numerical scheme. The properties of the proposed, explicit, finite-difference scheme are determined.

Nikos Karachalios (University of Thessaly)

The Closeness of the Ablowitz-Ladik Lattice to non-integrable DNLS Lattices

While the Ablowitz-Ladik (AL) lattice is integrable, the Discrete Nonlinear Schrödinger (DNLS) equation, which is more significant for physical applications, is not. We prove closeness of the solutions of both systems in the sense of a "continuous dependence" on their initial data in the suitable metrics. The most striking relevance of the analytical results is that small amplitude solutions of the Ablowitz-Ladik system persist in the DNLS systems. It is shown that the closeness results are also valid in higher dimensional lattices as well as for generalized, physically significant nonlinearities of the DNLS system. We also discuss extensions of this approach to NLS partial differential equations and its potential applications.

Anna Logioti (University of Bonn, Germany)

A non-local free boundary problem arising in a model of cell polarization

In this talk, I will present several results for a parabolic non-local free boundary problem that has been derived as a limit of a bulk-surface reaction diffusion system of equations which models cell polarization. In previous work we have justified the well-posedness of this problem and we have further proved uniqueness of solutions and global stability of steady states. Yet, these results were not sufficient in order to obtain much insight about the evolution of the support of the solution. Hence, in our recent work we investigate qualitative properties of the free boundary. In particular, we conclude that there are necessary and sufficient conditions for the initial data that yield continuity of the support. We show that whenever these assumptions fail, either jumps or (when restricted to the case of the unit sphere) fast oscillations for the support of the solution take place. In addition we provide a complete characterization of the jumps for a large class of initial data. (This is a joint work with Prof. B. Niethammer, Prof. M. Röger and Prof. J. J. L. Velázquez).

Leonidas Mindrinos (NTUA)

A numerical solution for various lateral Cauchy problems

We present a two-step method for the numerical solution of parabolic and hyperbolic Cauchy problems. The problems are formulated in two dimensions and the proposed method can be applied in both direct and inverse problems. The main idea is tocombine a semi-discretization in time combined with a boundary integral equation method for the spatial variables. The time discretization reduces the problem to a sequence of elliptic stationary problems. We describe the derived coefficients using a single-layer ansatz for some unknown boundary density functions. We solve the discretized problem on the boundary of the medium with the collocation method. Classical quadrature rules are applied for handling the kernel singularities. We present numerical results for different linear PDEs.

This research was carried out while I was a postdoctoral researcher at the University of Vienna, Austria. It is a joint work with R. Chapko (Ivan Franko University of Lviv, Ukraine) and B. T. Johansson (Linköping University, Sweden).

Georgios Moschidis (Princeton University, USA)

The emergence of weak turbulence in general relativity

The AdS instability conjecture provides an example of weak turbulence appearing in the dynamics of the Einstein equations in the presence of a negative cosmological constant. The conjecture claims the existence of arbitrarily small perturbations to the initial data of Anti-de Sitter spacetime which, under evolution by the vacuum Einstein equations with reflecting boundary conditions at conformal infinity, lead to the formation of black holes after sufficiently long time. In this talk, I will first introduce the setup of the initial-boundary value problem for the Einstein equations and discuss how non-trivial geometric features such as black holes can appear dynamically in the evolution of those equations. I will then present a rigorous proof of the AdS instability conjecture in the setting of the spherically symmetric Einstein-scalar field system. If time permits, I will also discuss possible paths for extending these ideas to the vacuum case.

Nikos Roidos (University of Patras)

The fractional porous medium equation on manifolds with conical singularities

We will talk about existence, uniqueness and maximal L^q-regularity results for solutions of the fractional porous medium equation on manifolds with conical singularities. We will also discuss the space asymptotic behavior of the solutions close to the singularities and its relation to the local geometry.

Georgios Sakellaris (Aristotle University of Thessaloniki)

Scale invariant regularity estimates for second order elliptic equations with lower order coefficients in optimal spaces

We will discuss local and global scale invariant regularity estimates for subsolutions and supersolutions to the equation $-\operatorname{div}(A\nabla u + bu) + c\nabla u + du = -\operatorname{div} f + g$, assuming that A is elliptic and bounded. In the setting of Lorentz spaces, under the assumptions $b, f \in L^{n,1}$, $d, g \in L^{\frac{n}{2},1}$ and $c \in L^{n,q}$ for $q \leq \infty$, we will see that, with the surprising exception of the reverse Moser estimate, estimates with "good" constants (that is, depending only on the norms of the coefficients) do not hold in general. On the other hand, assuming a necessary smallness condition on b, d or c, d, we will discuss a maximum principle and Moser's estimate for subsolutions with "good" constants. We will also see that the reverse Moser estimate for nonnegative superso- lutions with "good" constants always holds, under no smallness assumptions when $q < \infty$, leading to the Harnack inequality for nonnegative solutions and local continuity of solutions. Finally, in the setting of Lorentz spaces, we will show that our assumptions are the sharp ones to guarantee these estimates.

Konstantinos Tzirakis (IACM/FORTH & University of Crete)

Layer dynamics for the one-dimensional ε-dependent Cahn-Hilliard/Allen-Cahn equation

We study the dynamics of one-dimensional combined Cahn-Hilliard and Allen-Cahn models, within a neighborhood of an equilibrium of fixed N transition layers and examine its (meta)stability properties. We introduce two non-negative ε-dependent weights in order to control the coexistence of the two operators in terms of the "interaction length" ε which is small relative to the size of the sample, representing the layers' width. Two different settings are considered which differ in that, for the second, we impose a mass-conservation constraint in place of one of the zero-mass flux boundary conditions. We implement an N-dimensional, and a mass-conservative (N-1)-dimensional manifold respectively, where a metastable state with N transition layers is approximated. In the neighbor of the metastable state, we decompose the evolution into a component that is restricted to the approximate manifold and a component in an approximate "orthogonal" subspace. An important aspect of the study is an estimate of the spectrum of the linearized operator at the approximate metastable state. We then determine, for both cases, the essential dynamics of the layers (ode systems with the equations of motion), expressed in terms of the local coordinates relative to the manifold used. In particular, we specify wide families of the ε-dependent weights acting at each part of the operator, for which the dynamics are stable and rest exponentially small in ε. (This is a joint work with D. Antonopoulou and G. Karali).