Language: **English**

Thursday, June 21, 2018

Problem 1. Find all the pairs (m, n) of integers which satisfy the equation

$$m^5 - n^5 = 16mn.$$

Problem 2. Let n three-digit numbers satisfy the following properties:

(1) No number contains the digit 0.

(2) The sum of the digits of each number is 9.

(3) The units digits of any two numbers are different.

(4) The tens digits of any two numbers are different.

(5) The hundreds digits of any two numbers are different.

Find the largest possible value of n.

Problem 3. Let $k > 1$ be a positive integer and $n > 2018$ be an odd positive integer. The nonzero rational numbers $x_1, x_2, \ldots, x_n$ are not all equal and satisfy

$$x_1 + \frac{k}{x_2} = x_2 + \frac{k}{x_3} = x_3 + \frac{k}{x_4} = \cdots = x_{n-1} + \frac{k}{x_n} = x_n + \frac{k}{x_1}.$$

Find:

a) the product $x_1 x_2 \ldots x_n$ as a function of k and n

b) the least value of k, such that there exist n, $x_1, x_2, \ldots, x_n$ satisfying the given conditions.

Problem 4. Let ABC be an acute triangle, A', B' and C' be the reflections of the vertices A, B and C with respect to BC, CA, and AB, respectively, and let the circumcircles of triangles ABB' and ACC' meet again at A_1. Points B_1 and C_1 are defined similarly. Prove that the lines AA_1, BB_1 and CC_1 have a common point.

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Time: 4 hours and 30 minutes
Each problem is worth 10 points